J. Fluid Mech. (1970), vol. 42, part 4, pp. 755–768
Printed in Great Britain

Observed flow patterns at the initiation of convection in a horizontal liquid layer heated from below

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(Received 14 July 1969 and in revised form 11 March 1970)

An experimental investigation has been made of the flow patterns at the initiation of convection in a layer of a high Prandtl number liquid confined between rigid, horizontal surfaces and heated from below. The experiment was designed to overcome the limitations of earlier experiments and to correspond closely to the conditions of the theory. In particular, the upper and lower rigid surfaces which enclosed the layer were made of copper which has a high thermal conductivity. To aid in the analysis of the experimental results some supplementary observations of the flow patterns were made using a glass upper plate. For small fluid depths and large temperature differences between the upper and lower surface the initial flow was in the form of hexagonal cells as predicted theoretically. With increasing Rayleigh number the cellular flow appeared to transform into rolls as predicted. For large fluid depths and small temperature differences only circular plan-form rolls were observed. This is in agreement with the results of other experimenters. It is tentatively proposed that this non-appearance of an initial cellular flow is due to the shape of the test chamber having a dominating influence on the flow pattern when the temperature gradient in the fluid is small. Measurements were also made of the development time for the flow patterns and the critical Rayleigh number.

1. Introduction

The tendency for fluid motions to commence in a horizontal heated fluid layer can be analyzed using linear methods and assuming temperature-independent fluid properties [see Chandrasekhar (1961) for a complete discussion]. It is found that the initiation of motion requires conditions in the fluid layer to attain a critical state which can be represented by a dimensionless parameter, the Rayleigh number (Ra), defined in the caption to figure 1.

At Rayleigh numbers slightly above critical the results of linear calculations are not in agreement with experimental observations nor are they able to give much indication of how particular flow patterns are formed. Thus, according to the linear calculations, disturbances associated with a band of wave-numbers will be unstable and they are predicted to grow without bound. Experiment shows that the disturbances combine in certain ways and they do not continue

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to grow but attain a finite steady amplitude. To obtain agreement between theory and experiment non-linear effects must be considered.

Theoretical calculations have also shown that asymmetries in the heated fluid layer (see Palm 1960; Segal 1966, p. 165), as well as non-linear effects, are an important mechanism in the formation of the supercritical flow pattern. The combined effect of non-linearities and of asymmetries due to fluid property variations between the upper and lower surfaces has been investigated theoretically by Busse (1962), Segel (1965) and Palm, Ellingsen & Gjevik (1967). They have shown that the initial flow pattern in a fluid layer of infinite horizontal extent confined between rigid horizontal surfaces consists of cells with a hexagonal plan form. This flow pattern is predicted to be stable for a range of Rayleigh numbers but transforms into a two-dimensional roll pattern at some supercritical Rayleigh number. Krishnamurti (1968*a, b*) has shown that a hexagonal cellular flow pattern results from time-dependent heating or cooling of the fluid layer, which produces an asymmetry in the temperature profile.

Experimental support for the theoretical predictions is rather uncertain. Thus, although Silveston (1958) was able to show that the initial motion was always cellular, in Koschmieder's (1966) experiments, on the other hand, the flow always commenced in the form of circular rolls and this persisted as the Rayleigh number increased. These experiments are analyzed in detail in the next section and it is shown that the conditions were probably not suitable for a satisfactory test of theoretical predictions.

In order to provide a more certain evaluation of the theory (Busse 1962) we have undertaken an experimental investigation of the flow patterns that appear for slightly supercritical values of the Rayleigh number under steady conditions with fluid property variations. The methods used in the analysis of the earlier experiments were employed to ensure that the apparatus was designed, and the experiments conducted, in such a way that conditions were optimized for the appearance of the predicted cellular flow pattern. These analytical techniques were also used to interpret our experimental observations.

2. Discussion of previous experiments

A number of quantitative experimental investigations of the flow in a horizontal, heated fluid layer have been reported. Both gases and liquids have been used in these earlier experiments but experimental convenience has restricted the experiments discussed in this paper to the use of high Prandtl number liquids. In these circumstances it is proposed to review only the earlier experiments in which liquids were employed.

The analysis is based on figure 1 which is a diagnostic diagram showing the effect of fluid property variations (as represented by the parameter R_{11}, which is defined in the caption of figure 1) on the flow patterns.† The figure has been

† Although it is possible to plan and interpret the experiments with the aid of figure 1, this diagram must be used cautiously because the theoretical calculations are approximate so the shapes of the curves separating the different flow régimes may not be of the exact form shown.

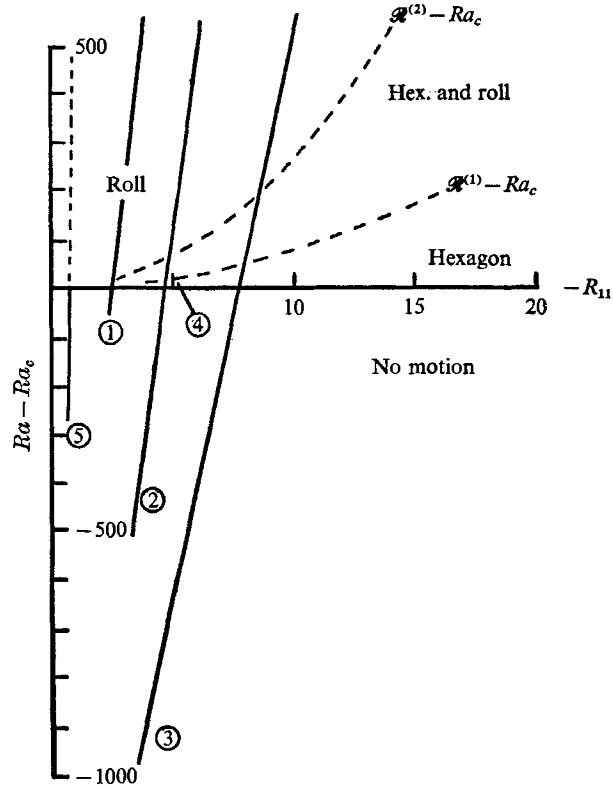

FIGURE 1. Diagnostic diagram for free convection in a heated, horizontal layer of fluid of infinite Prandtl number and with rigid–rigid boundary conditions. Numbered curves represent results of Silveston and Koschmieder (see table below)

$$R_{11} = 15{\cdot}28\left(\frac{\nu_H-\nu_C}{\nu_M}\right)+14{\cdot}85\left(\frac{\rho_c-\rho_H}{\rho_M}\right)-34{\cdot}55\left(\frac{C_H-C_c}{C_M}\right)+16{\cdot}18\left(\frac{k_H-k_c}{k_M}\right),$$

where Ra = Rayleigh number $= g\beta\rho CL^3\Delta T/\nu k$ where g = acceleration of gravity, β = coefficient of volume expansion, ρ = density, C = specific heat, L = distance between upper and lower surfaces of the fluid layer, $\Delta T = T_H - T_c$, T_H = hot (lower) plate temperature, T_c = cold (upper) plate temperature, ν = kinematic viscosity, k = thermal conductivity. Ra_c = critical Rayleigh number at which fluid motion commences value of Jenssen (1963) used.

$\mathscr{R}^{(1)}$ = critical Rayleigh number below which only cellular motions are stable $= Ra_c + 0{\cdot}80R_{11}^2$,
$\mathscr{R}^{(2)}$ = critical Rayleigh number above which only roll motions are stable $= Ra_c + 2{\cdot}75R_{11}^2$.

Investigator	Curve	L (cm)	Mean fluid temperature (°C)	Fluid	Prandtl number	Notes
Silveston	1	1·000	30	AK350	3000	From steady-state heat transfer data with copper upper surface
Silveston	2†	0·698	50	AK350	3000	
Silveston	3†	0·698	30	AK350	3000	
Koschmieder	4	0·515	23·4	DC200–100	900	From given data and nominal fluid properties
Koschmieder	5	1·015	23·4	DC200–100	900	

† Not clear from the original reference which of these would correspond to the observed flow patterns.

constructed from the calculations of Busse (1962) as presented in a convenient form by Davis & Segel (1968). In constructing figure 1 the theory of Jenssen (1963) has been used to determine the variation of the critical Rayleigh number (Ra_c) due to variation in the fluid viscosity with temperature. The experiments of Silveston and Koschmieder are represented on figure 1 by the appropriately numbered curves.

In Silveston's experiments the initial flow pattern was always cellular. At some supercritical Rayleigh number this flow was replaced by (straight?) rolls. This is qualitatively in agreement with the theoretical predictions shown in figure 1. However, comparison with the theory is uncertain. The method of cooling (free convection to the ambient) may have produced an uneven temperature distribution on the poorly conducting glass upper surface of the test chamber, so that conditions did not correspond to the uniform plate temperature assumed in the theory. These experiments were also unsteady in nature and the speed with which the Rayleigh number changed (as high as 70 per minute in one case) may have caused the observed cellular flow pattern to be of the type predicted by Krishnamurti.

Flow commenced below the theoretical critical Rayleigh number in Koschmieder's experiments with the appearance of circular rolls at the circular side walls of the test chamber. As the Rayleigh number increased more rolls appeared and these propagated toward the centre of the test chamber until at the critical Rayleigh number the whole chamber was filled with concentric rolls. These rolls persisted as the Rayleigh number increased.

An examination of figure 1 suggests that Koschmieder may not have conducted his experiments under the optimum conditions for the appearance of cellular flow. It can be seen that for small values of the property variation parameter (R_{11}) the hexagonal band is so narrow that it would be difficult to obtain sufficiently close control over the experimental parameters (particularly the temperature difference between the hot and cold surfaces) so as to ensure that experimental conditions correspond to a point located in the hexagonal band.

It is also possible that Koschmieder changed the experimental conditions too rapidly for the predicted flow pattern to develop. Segel (private communication) has estimated that the time for rolls to grow from 0·1 to 0·9 of their final amplitude at a given Rayleigh number is inversely proportional to the difference between that Rayleigh number and the critical Rayleigh number. A similar result is apparently not available for the growth time of hexagonal cells. However, if it is supposed that this rule is applicable to a cellular flow pattern, then it follows that the narrower the hexagon stable band in figure 1 the longer it will take for the flow to develop. Thus in quasi-steady experiments at low values of R_{11}, such as those conducted by Koschmieder, it is essential to change the experimental conditions sufficiently slowly to allow the stable flow pattern to develop.

Figure 1 is based on the assumption that property variation effects dominate other factors which may influence the flow pattern (such as the test chamber shape or asymmetries due to time-dependent heating). In Koschmieder's experiments the property variation effect (R_{11}) was small so that effects due to test chamber shape may be dominant (as concluded by Koschmieder). This could be the cause of the supercritical flow pattern being in the form of circular rolls.

Clearly, the experimental evidence for the existence of a cellular flow pattern is not strong. However, the experiments of Krishnamurti (1968*b*) in which such a flow was produced by time dependent heating are encouraging because they demonstrate that at least one type of asymmetry does, in fact, produce a cellular flow. Under these circumstances it would seem appropriate to conduct a series of experiments concerned with the initial flow in a heated fluid layer. However, the analysis of the experiments of Silveston and Koschmieder suggests that to obtain the predicted cellular flow pattern, it is necessary for a variety of reasons, to conduct the experiments at large values of the property variation parameter (R_{11}) and to change the system temperatures very slowly.

3. Apparatus and experimental procedure

A brief description will be given of the experimental apparatus and procedure. More extensive discussions will be found in Somerscales & Dougherty (1969) and Dougherty (1970).

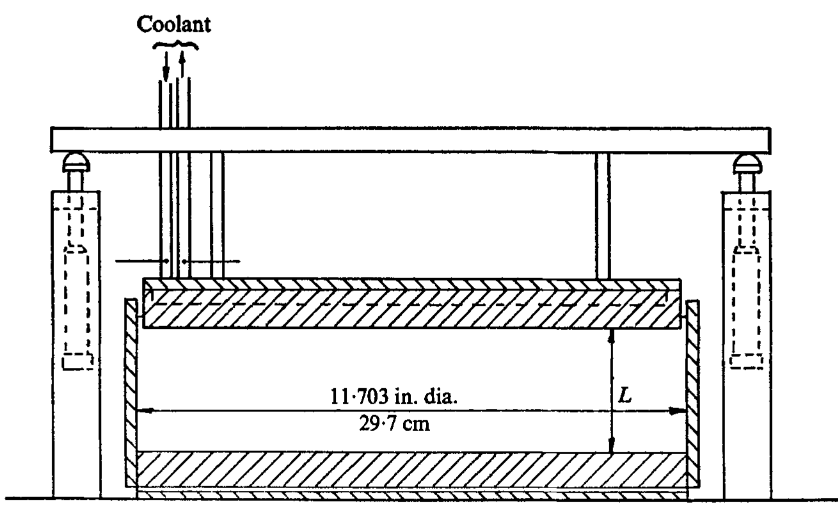

FIGURE 2. Test chamber with copper upper plate.

The apparatus used in these experiments is shown in figure 2. It consisted of a circular plan-form container with an electrically heated copper plate at the bottom and Plexiglas side walls. The apparatus shown in figure 2 has a copper upper surface but, as mentioned later, in some experiments the copper plate was replaced by a water-cooled, glass plate. The upper plates (glass or copper) were carefully designed to minimize temperature non-uniformities. The upper plate was suspended from a triangular frame which was supported at its corners by vertically mounted micrometer screws. There was a small clearance (0·159 cm) between the upper plate and the chamber side walls so that the spacing between the upper and lower surfaces could be easily varied.

The test chamber was mounted on an insulating support which could be levelled and which rested on vibration isolators. The apparatus was enclosed by polyurethane foam to provide heat insulation.

The plate temperatures were measured by calibrated copper-constantan thermocouples. The rate of heat transfer through the fluid layer could be determined either from measurements of the power input to the electrical heater, by using the copper upper plate as a calorimeter or by means of Boelter heat flow meters (Jakob 1957) in the lower plate.

The top plate cooling water temperature was maintained within 0·06 °C by a constant temperature circulator. The temperature controller on the circulator could be programmed to change the upper plate temperature at a rate of 0·139 °C per hour. The lower plate temperature was held constant at a predetermined value by means of a d.c. power supply connected to the lower plate heating element. This power supply was automatically controlled so as to maintain a particular lower plate temperature.

Dow Corning 200 silicone fluid with a nominal viscosity at 25 °C of 1000 centistokes was used as the experimental fluid. The properties of this fluid are given in the reports of Somerscales & Dougherty (1969) and Dougherty (1970).

The photographic technique of Krishnamurti (1967, 1968*b*) was used to obtain plan views of the flow pattern when using the copper upper surface on the test chamber. For this purpose a small quantity (0·02 % by weight) of aluminium dust was added to the fluid to make the flow pattern visible. Calculations and measurements indicated that the effect of this on the fluid properties was negligible.

It was found that the variation with temperature of the fluid index of refraction limited the effectiveness of the Krishnamurti technique. Both the incident light beam and the light scattered from the aluminium particles were subject to deviations so that points in the fluid farthest from the camera were not adequately photographed. Although it was our primary intention to examine the flow between metal plates of high conductivity, so as to duplicate the theoretical conditions as closely as possible, it was decided to make supplementary observations using a glass upper plate. This allowed the flow patterns to be viewed directly. The runs with the glass upper surface duplicated the conditions with the copper plate and the corresponding photographs were correlated.

On completion of thermocouple calibration the apparatus was levelled and assembled. The supporting micrometers were carefully calibrated in terms of plate spacing [see Somerscales & Dougherty (1969) and Dougherty (1970) for details].

Initially the apparatus was maintained at a uniform temperature by circulating water through the upper plate with no power input to the lower plate heater. The lower plate was then heated to its operating temperature, as set on the automatic controller, in about 30 min. Once the system temperatures had steadied the run was commenced.

Two different types of experiments were conducted. In the so-called quasi-steady experiments the upper plate temperature was decreased at a rate of 0·139 °C per hour with the bottom plate held at a constant temperature. To investigate the growth time of the flow patterns and the effect of the temperature–time history, subcritical flows (see §4) and air bubbles in the cooling water (see §4), so-called rapid heating to steady state runs were performed. In these runs the upper and lower plate temperatures were held constant after the operating conditions were attained.

During the course of a run the development of the flow patterns was observed. Data were taken at about every 0·55 °C change in upper plate temperature in the quasi-steady runs.

The experimental conditions are summarized in table 1 and figure 3. The results of representative runs are discussed in §§4, 5 and 6.

Run number	Upper plate	Curve number in figure 3	Rayleigh number range	L (cm)	T_H (°C)	T_c (°C)
2†	Copper	2	1430–2040	0·940	48·9	35·0–25·6
3†	Copper	3	950–2040	0·825	48·9	35·0–12·9
4†	Copper	4	1270–2040	0·725	60·0	35·0–11·5
5‡	Copper	4	1700	0·725	60·0	23·9
6†	Glass	4	1530–1920	0·725	60·0	24·4–16·3
*AB*1‡	Glass	2	2100	0·940	48·9	27·0
*AB*2‡	Glass	4	1680	0·725	60·0	24·5
7‡	Glass	4	1720	0·725	60·0	23·0

† Quasi-steady run, the system was rapidly heated to the initial Rayleigh number and then the upper plate temperature was decreased at 0·139 °C/h. No photographs were taken until the system temperatures had stabilized.

‡ Rapid heating to steady state, as indicated by the Rayleigh number. Photographs were taken during rapid heating and at steady state.

TABLE 1. Experimental conditions

4. Flow patterns

In run 2 the Rayleigh number was held at an initial value of 1430. At the end of 110 h four boundary rolls were present together with a weak straight line motion in the centre of the fluid layer (figure 4(*a*), plate 1). The origin of this flow was not clear but it is discussed in more detail in §7. The boundary rolls propagated to the centre of the fluid layer as the Rayleigh number increased and completely filled the test chamber at a Rayleigh number of 1740 ($= 1{\cdot}02\,Ra_c$). It could be seen (figure 4(*b*), plate 1) that where the circular and straight rolls intersected perpendicularly to one another there was an apparent cellular flow. In view of the results obtained in subsequent runs (particularly run 6) this was believed to be due to the interaction of the straight and circular roll patterns and was not the predicted cellular flow pattern caused by the variation of fluid properties with temperature.

The initial flow patterns in run 3 were similar to those in run 2 except the straight roll motion in the centre appeared at a Rayleigh number of about 1200. At a Rayleigh number of 1760 (figure 5, plate 2†) it appeared as if the straight

† See p. 762.

rolls were 'pinching off' (see figure 8 (*a*), plate 4 for an example of this with circular rolls). The limitations of the photographic technique did not allow a more definite conclusion to be drawn. It should be noted that the corresponding data point lies in the hexagon band of the diagnostic diagram (figure 3). When the Rayleigh number increased to 1780 ($= 1{\cdot}06\,Ra_c$) the circular rolls filled the test chamber.

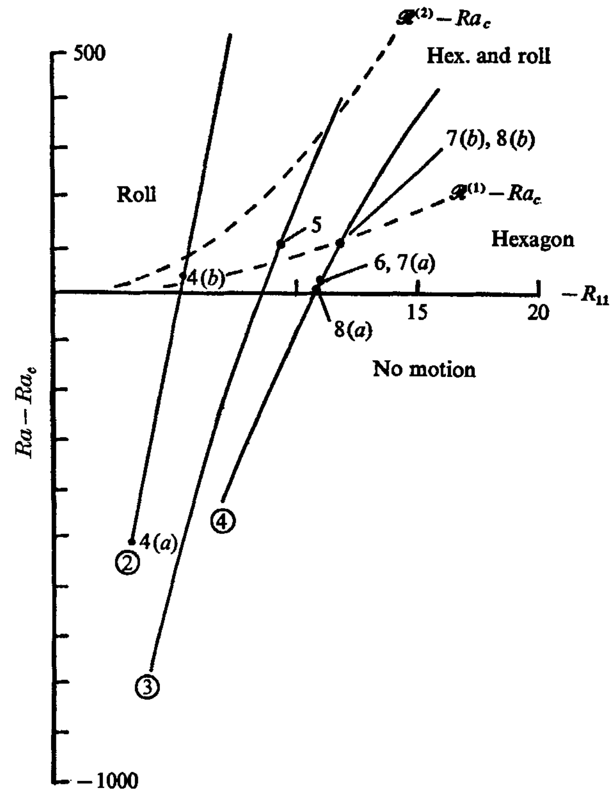

FIGURE 3. Diagnostic diagram for free convection in a heated, horizontal layer of fluid of infinite Prandtl number and with rigid–rigid boundary conditions. Numbered curves correspond to different plate spacings (see table 1). Points corresponding to figures 4 to 8 are also shown. R_{11} defined in the caption for figure 1.

As before, rolls appeared at the sidewall in run 4 and propagated toward the centre as the Rayleigh number increased. At a Rayleigh number of 1670 (figure 7 (*a*), plate 3) the core enclosed by the circular rolls appeared to consist entirely of cells. The transition from straight line rolls in the core to a cellular motion had occurred between Rayleigh numbers of 1630 and 1670. This pattern persisted as the hexagon stable band was passed through but at the boundary between the hexagon and hexagon and roll regions in the flow pattern appeared to be transforming into a system of concentric rolls (figure 7 (*b*), plate 3). The possibility of

† Although this figure and figure 7 are rather indistinct, they are included in the paper because the primary objective of these experiments was to make observations of the flow patterns between high conductivity copper plates (as opposed to low conductivity glass plates, such as in figures 6 and 8). Furthermore, so far as we are aware, they represent the only observations of cellular flow due to fluid property variations under such conditions.

this cellular flow being caused by interaction between the straight and circular rolls is discussed below.

Run 6 was a duplicate of run 4 using a glass upper plate on the test chamber (for the reasons explained earlier). At a Rayleigh number of 1530 four (or perhaps five) circular rolls were present at the boundary with no motion in the central core of fluid. Some motion commenced in the centre at a Rayleigh number of 1540 and at a Rayleigh number of 1670 ($= 0{\cdot}991\, Ra_c$) a cellular flow was clearly visible in the central core. When the Rayleigh number reached 1770 ($= 1{\cdot}07\, Ra_c$) (see figure 8 (*b*), plate 4) the cellular flow pattern occupied almost the entire extent of the test chamber. The circular rolls had apparently 'pinched off' at the critical Rayleigh number to form a system of hexagonal cells arranged in a circular pattern (see figure 8 (*a*)). This flow persisted until the run was terminated (by the limitations of the experimental equipment) at a Rayleigh number of 1920 (which lies in the band between $\mathscr{R}^{(2)} - Ra_c$ and $\mathscr{R}^{(1)} - Ra_c$). It should be noted that figures 7 (*a*), 8 (*a*) and figures 7 (*b*), 8 (*b*) were taken at about the same Rayleigh numbers, 1670 and 1760, respectively. This will provide some comparison of the flow patterns observed with the copper and the glass upper surfaces.

In run 6 it was found that air dissolved in the cooling water, came out of solution and collected as bubbles on the glass plate (see figures 8 (*a*) and 8 (*b*)). It was considered possible that these would produce a non-uniform temperature distribution on the upper plate. The bubbles also tended to form a star-shaped pattern which probably indicated that the flow distribution over the glass plate was not uniform. If the upper plate has a non-uniform temperature distribution the experimental conditions do not correspond to the theoretical assumptions. Furthermore, the observed flow pattern may be a consequence of this condition and not be due to the variation of fluid properties between the upper and lower plates. To check the effect of these uncertainties on the experimental results the following tests were conducted: the system was rapidly heated in one test to a point in the roll stable region (run $AB1$) and in another case to a point in the hexagon stable region (run $AB2$). The flow pattern was observed for 72 h to see if the accumulating air bubbles caused the flow pattern to undergo a transformation. It was found that: (i) In run $AB1$ the roll pattern did not change into a cellular flow pattern. (ii) In run $AB2$ cells initially only appeared in the vicinity of an air bubble, the fluid being otherwise at rest. But at a later time (figure 6) there were cells in the vicinity of air bubbles but there were also cells which were not in the vicinity of air bubbles.

It is concluded that where hexagons are not predicted to be stable the temperature perturbations in the upper plate associated with the air bubbles in the cooling water do not cause a hexagonal cellular flow pattern to appear.† It is further concluded that where hexagons are predicted to be stable the upper surface temperature perturbations may cause a cellular flow to appear but it is not an essential condition. In fact, in the absence of upper surface temperature perturbations, i.e. when using a copper plate (run 4), the hexagonal cellular flow pattern still appeared when it was predicted to be stable (figure 7 (*a*)).

† This could also be seen under subcritical conditions.

5. Development time of the flow patterns

It was mentioned in §2 that the flow pattern growth time may vary inversely as the width of the hexagon band in figure 1. This was investigated in runs 5 and 7 which duplicated the conditions of run 4 except that the system was rapidly heated to a point in the hexagonal band and held at that condition for many hours. In both runs the initial flow pattern consisted of boundary rolls that propagated to the centre of the test chamber as time went on. A transformation to a hexagonal cellular flow pattern started in run 5 about 48 h after reaching steady-state conditions and in about half this time in run 7. Since the super-criticality condition ($Ra - Ra_c$) in run 7 was about twice that in run 5 this is in agreement with the criterion presented above. The cellular flow pattern was fully developed after 72 h in run 5 and 36 h in run 7.

The initial appearance of the boundary rolls in run 7 occurred twice as rapidly as in run 5. Again, this is in accordance with Segel's criterion. However, the individual development times were about 10 times greater than predicted. It seems probable that these boundary rolls are not of the type discussed by Segel (private communication) (the theory assumes a layer of infinite horizontal extent). Consequently these boundary rolls cannot be expected to conform to the theoretical predictions.

6. The critical Rayleigh number

The heat transfer measurements, the output of the heat flow meters and observations of the flow patterns (the photographs) were used as three different means of ascertaining the critical Rayleigh number. The use of the heat transfer measurements and heat flow meter readings was straightforward. They were plotted against Rayleigh number and it was then found possible to draw, by eye, two straight lines through these data. The point of intersection was taken to be the critical Rayleigh number. The flow pattern photographs did not provide such a clear cut criterion. The critical Rayleigh number was considered to occur when the observed flow pattern changed from subcritical straight rolls to either a system of concentric rolls, completely filling the test chamber, or to a flow pattern made up of hexagonal cells surrounded by boundary rolls. Because it was not possible to obtain a continuous series of flow pattern photographs in the vicinity of the critical conditions it was only possible to define a range of Rayleigh numbers within which the critical value lies.

The experimental critical Rayleigh numbers are compared with the predictions of Jenssen (1963) in table 2. The lower and higher Rayleigh numbers shown in the second column correspond, respectively, to the highest and lowest Rayleigh numbers at which supercritical flow patterns were absent or present. The average of these values agrees within about $\pm 2\%$ with Jenssen's results and exhibits the predicted decrease in critical Rayleigh number with increasing fluid property variation. The critical Rayleigh numbers obtained by the other methods (the fourth and fifth columns in table 2) are generally higher than those obtained photographically because the subcritical Nusselt number was greater than unity.

This was caused by the subcritical motions (boundary rolls and straight rolls) increasing the heat transfer over that due to pure conduction (unity Nusselt number).

Run number	Means of detecting the critical Rayleigh number: Flow patterns — Range	Flow patterns — Average	Heat transfer measurements	Heat flow meters	Jenssen's critical Rayleigh number‡
2	1700–1740	1720	1710	Not measured	1698
3	1630–1690	1660	1800	1650	1687
4	1630–1680	1650	1820	1760	1665
6†	1640–1680	1660	1700–1800	1730	1665

† Due to scatter of data the critical Rayleigh number could only be determined to lie in the range shown under 'Heat transfer measurements'.

‡ This is calculated assuming that only the fluid viscosity varies with temperature. However, since in the fluid used in the experiments reported the only property having a marked variation with temperature is viscosity the error is less than the uncertainty of the experimental values of the critical Rayleigh number.

TABLE 2. Comparison of theoretical and measured critical Rayleigh numbers

7. Error considerations

The measured values of the temperature difference between the upper and lower surfaces were estimated to have an uncertainty between 1·25 % (for the minimum temperature difference) and 0·5 % (for the maximum temperature difference). The uncertainty in the plate spacing is considered to be 0·88 % (for the minimum spacing) and 0·68 % (for the maximum spacing).

In runs 2, 3 and 4 (where the copper upper surface was used) an initial subcritical flow pattern consisting of concentric rolls at the side walls and a straight roll motion was observed. The circular rolls were anticipated in view of the similar flow pattern obtained by Koschmieder but apparently he did not observe the straight rolls. The axes of the rolls were aligned at about 20 degrees to the axis of the lower plate heating element. This flow could be caused by (i) upper and lower plates not being parallel; (ii) non-uniformity in plate temperature; (iii) a genuine subcritical flow pattern.

This flow pattern was found to be insensitive to changes in the slope of the upper plate, item (i). The validity of the other causes were not investigated but non-uniform plate temperature would appear to be most likely. However, the hot and cold (copper) surface temperature uniformity was such that the average variation amongst the plate thermocouples was $\pm 0{\cdot}06$ °C. The motion was very weak, as indicated by its effect on the magnitude of the heat transfer. Together with the circular rolls at the side walls it caused about a 5 % increase in the Nusselt number. In view of this, its effect on the stable flow patterns was considered to be negligible.

It is possible that the cellular flow pattern originated from the straight rolls (see figures 4 (*b*) and 7 (*a*)). To test this hypothesis the system was rapidly heated from zero Rayleigh number to a point in the cell stable region (run 5). The straight line flow pattern did not develop but, as mentioned in § 5, a cellular flow pattern did develop from a circular roll pattern. It is, therefore, concluded that the cellular flow pattern is not associated with the subcritical straight rolls.

In a fluid layer in which the propagation of changes in boundary surface temperature is slow compared to the rate of change of surface temperature, the temperature distribution in the fluid will tend to become non-linear. A hexagonal cellular flow of the Krishnamurti type may consequently develop. To ensure that this does not occur, effects due to fluid property variations must dominate the effects due to the rate of change of mean fluid temperature. The value of the temperature change parameter ($R^{11} = 200\,L^2(\partial T_c/\partial\theta)/2\alpha\Delta T$ where θ = time, α = thermal diffusivity, all other quantities are defined in the caption to figure 1) varies from a value of $9{\cdot}5 \times 10^{-2}$ in run 2 to $2{\cdot}8 \times 10^{-2}$ in runs 4 and 6. The ratio of the property variation parameter (R_{11}) at the critical Rayleigh number (see figure 4) to the temperature change parameter is 53 in run 2 and 360 in runs 4 and 6. So for the experiments reported in this paper the property variation effects are more than 50 times larger than the effects due to the changing mean temperature.

A preliminary estimate of the Nusselt number uncertainty using the electrical power input corrected for heat losses was 4 %. This estimate was checked by making measurements at a very low subcritical Rayleigh number (where the Nusselt number should be unity) and the uncertainty in the measurement was found to be about 5 %. The heat loss determination was therefore considered satisfactory and the Nusselt number was assumed to be accurate within ± 4 %. The Rayleigh number was estimated to be accurate within 5 %.

8. Conclusions

It was found that for a sufficiently great fluid property variation with temperature (as indicated by the property variation parameter R_{11}) the initial supercritical flow pattern was a system of hexagonal cells as predicted by the calculations of Busse (1962), Segel (1965) and Palm *et al.* (1967). But prior to the appearance of the hexagonal flow a system of subcritical, circular rolls, of the type observed by Koschmieder (1966), appeared at the side walls of the test chamber. The cellular flow originated in the fluid enclosed by the circular rolls. The rolls then transformed into cells so that the resulting flow pattern consisted entirely of hexagonal cells. The roll motion is probably due to unavoidable edge effects in any practical experimental apparatus. For smaller values of the property variation parameter no cellular flow appeared.

These results, together with the observations made by Krishnamurti (1967) on the effect of time-dependent heating, suggest that the shape of the test chamber and the effects of asymmetries due to property variations and time-dependent heating all play a part in determining the initial flow pattern. However, one of these (in our case, property variation effects) can be made sufficiently large to dominate the others and therefore control the formation of the flow pattern.

Therefore it is tentatively proposed that, with small time-dependent heating effects, the initial supercritical flow pattern for small values of the property variation parameter is in the form of circular rolls for a test chamber of circular plan-form but at large values ($|R_{11}| \gtrsim 8$) the initial flow is in the form of hexagonal cells.

In these circumstances, figure 1 may not represent the experimental conditions for all values of the property variation parameter. Thus, it may be necessary to modify this diagram to take this into account, as has been proposed by Segel (1969).

The cellular flow appeared to convert into a system of circular rolls at some supercritical Rayleigh number. However, this conclusion is rather uncertain because the quality of the photographs made them difficult to interpret and because a similar effect was not seen when the copper upper plate was replaced with a glass plate.

The necessity for allowing sufficient time (as long as 72 h, in one case) for the development of the flow pattern was clearly demonstrated. This may, in part, be the reason why Koschmieder did not observe a cellular flow pattern since in his experiments the Rayleigh number was changed from zero to about five times critical in 8 to 10 h.

The critical Rayleigh number was found to decrease with increasing property variation as predicted by Jenssen (1963).

The investigation reported here is based on a dissertation presented by T. S. Dougherty to the faculty of Rensselaer Polytechnic Institute in partial fulfilment of the requirements for the degree of Doctor of Philosophy. The award of a NASA Traineeship to one of us (T. S. D) is gratefully acknowledged.

The authors are especially grateful to L. A. Segel for helpful discussions during the course of this work. We are also indebted to E. L. Koschmieder for valuable criticism and advice and to P. L. Silveston for discussing certain aspects of his experiments with us.

We are also grateful to the following organizations and individuals for assistance: U.S. Army, Watervliet Arsenal, W. F. Wells (deceased), J. Marshall, D. Shaver, K. E. Bisshopp, L. S. Coonley, S. E. Wiberley, M. Avila, and R. R. Buch.

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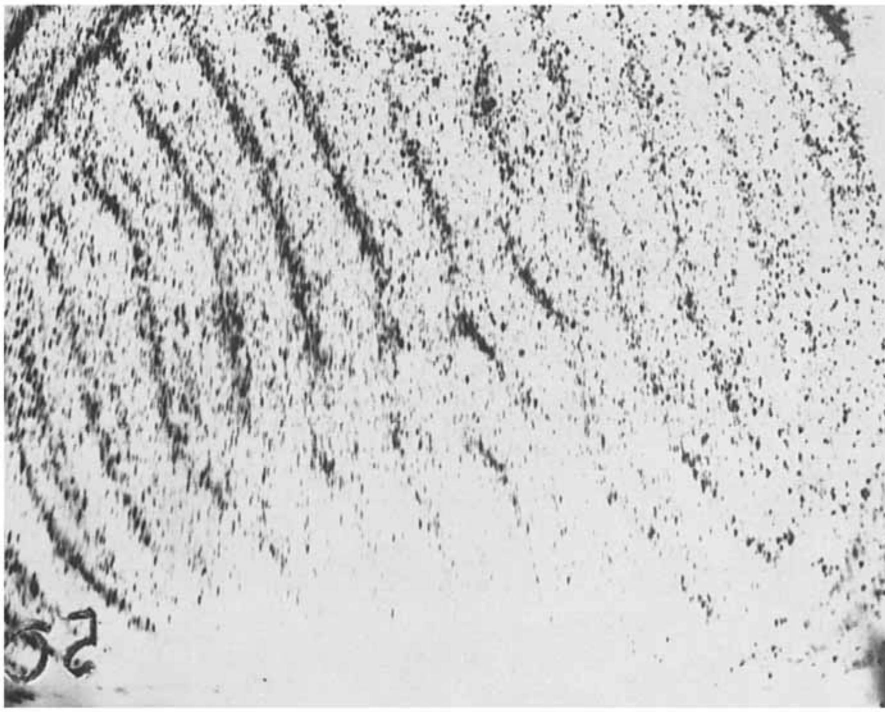
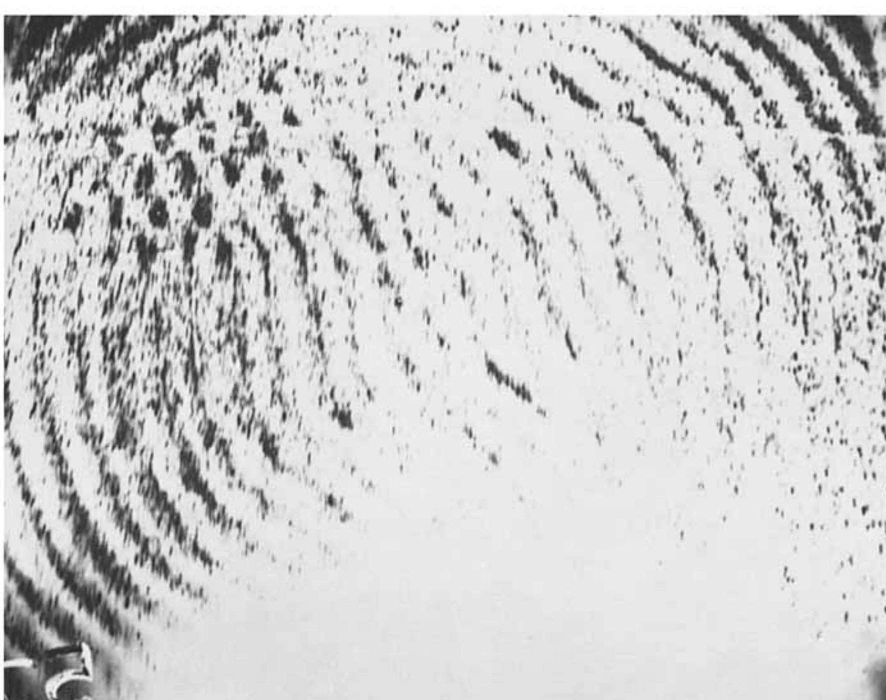


(*a*)



(*b*)

FIGURE 4. Flow patterns in run 2. (*a*) Four circular rolls and straight rolls. $Ra = 1430 = 0{\cdot}840Ra_c$. (*b*) Ten circular rolls and the intersection of circular and straight rolls. $Ra = 1740 = 1{\cdot}02Ra_c$.

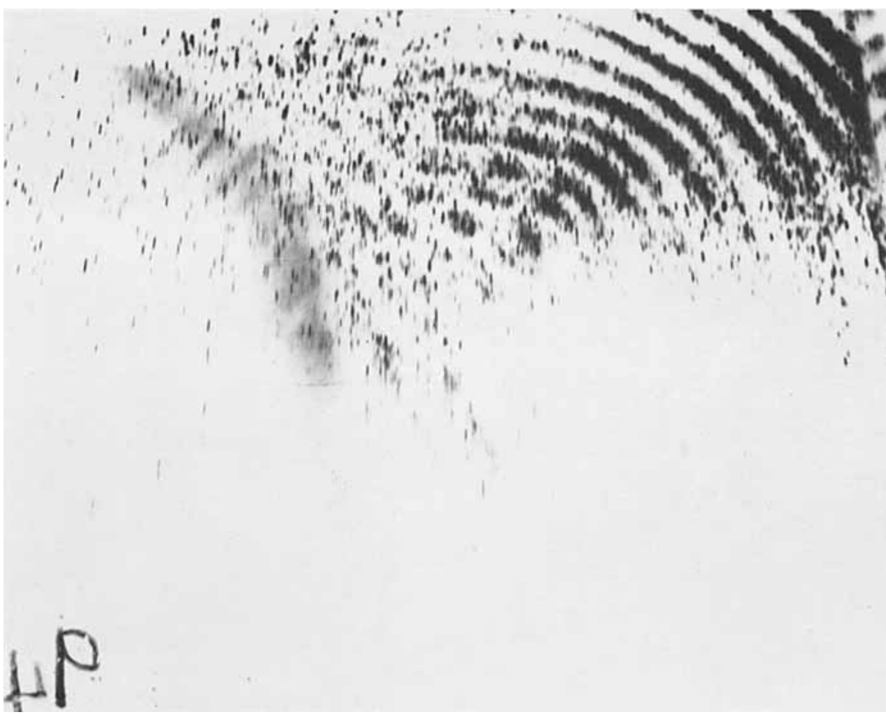


FIGURE 5. Flow pattern in run 3. Ten circular rolls and straight rolls 'pinching off'. $Ra = 1760 = 1{\cdot}05Ra_c$.

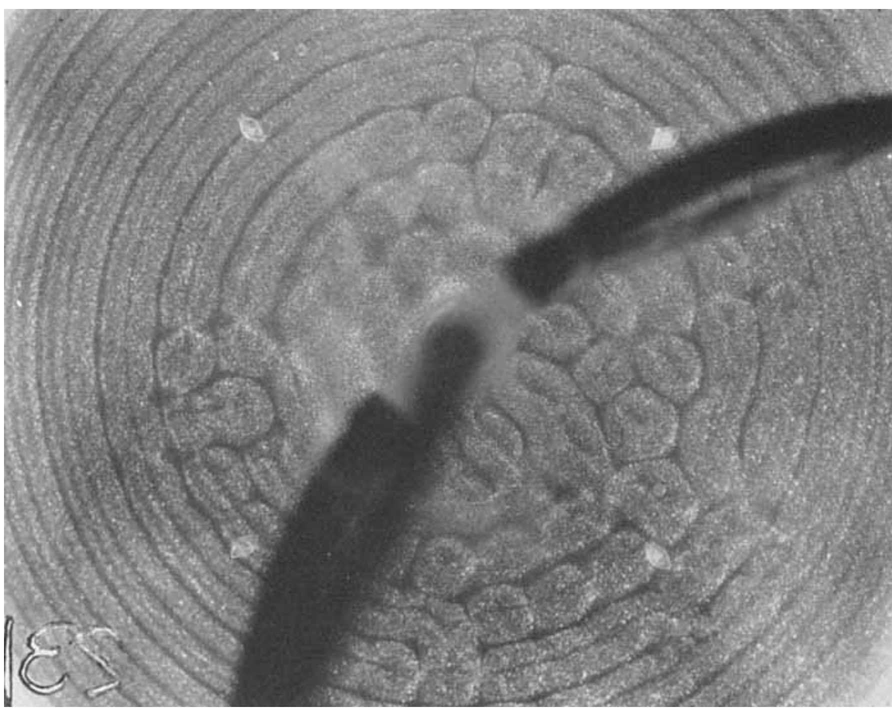
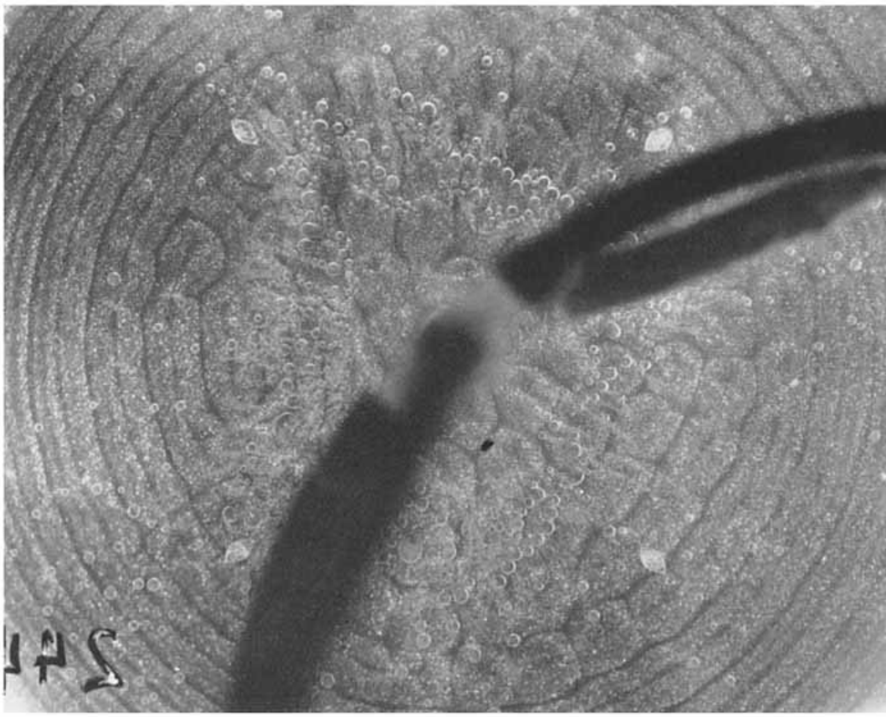


FIGURE 6. Flow pattern in run *AB*2. Showing that cells may or may not appear in the vicinity of air bubbles. $Ra = 1680 = 1{\cdot}01 Ra_c$.

(*a*)

(*b*)

FIGURE 7. Flow patterns in run 4. (*a*) Six circular rolls enclosing a possible cellular flow pattern. $Ra = 1670 = Ra_c$. (*b*) Cellular flow pattern transforming into circular rolls. $Ra = 1760 = 1{\cdot}06Ra_c$.



(*a*)

(*b*)

FIGURE 8. Flow patterns in run 6. (*a*) Circular rolls 'pinching off' to form a cellular flow pattern. $Ra = 1670 = Ra_c$. (*b*) Cellular flow pattern occupying almost the entire extent of the test chamber. $Ra = 1770 = 1{\cdot}07Ra_c$.